第五屆全港小學數學挑戰賽(2018-2019)
The 5th Hong Kong Primary Mathematics Challenge (2018-2019)

決賽 (二零一九年三月三十日)
Final (30th March, 2019)

小六組 Primary 6　　組別項目 Group Event　　試卷 Question Paper

參賽者須知 Instructions to Contestants

1. 在比賽過程中，參賽者必須將准考證放在桌上。
   You should place your Admission Form on your desk for the whole session.
2. 於比賽期間必須關掉所有手提電話、通訊工具及其他響鬧裝置。
   During the competition, you should switch off your mobile phone and any other electronic or communication devices that can emit sound.
3. 本項目以筆試形式舉行，須於限時 45 分鐘內完成所有題目。
   Contestants should finish all questions in this 45-minutes written test.
4. 在答題紙上填寫學校名稱、參賽者姓名及班級、參賽者編號、座位編號。
   Write your name, class, admission number, seat number and school name on the front cover of your answer sheet.
5. 參賽者於比賽時只准使用大會提供之草稿紙。
   You can only use the rough work sheet provided by the organizer.
6. 參賽者不可於比賽中使用計算機。
   The use of calculators is NOT allowed.
7. 每題只需把答案填寫在大會提供之答題紙上，否則不予評分。參賽者不需填寫計算步驟。
   Put your answers on the answer sheet provided, otherwise, the answers will not be marked. You are not required to show the steps in your calculations.
8. 除非問題特別聲明，分數的答案須化至最簡。
   Unless otherwise stated by the question, answers of fraction should be expressed in their simplest form.
9. 除特殊情況外，參賽者於本項目完結前不能提早交卷或離場。
   Under normal circumstances, contestants are not allowed to leave the contest venue before the end of this session.
10. 違反比賽規則者有可能被取消參賽資格。
    Any contestant who violates the rules and regulations of the competition might risk disqualification.
11. 參賽者如對比賽過程或試題內容有任何疑問或爭議，參賽者須於當天比賽結束後立即向大會提出，否則不予受理。大會保留是次比賽的所有最終決定權。
    If you have any queries, you should contact the officer-in-charge immediately after the competition. Late queries will not be entertained. The decision of the organizing committee will be final.

時限：四十五分鐘　　總分：400
Time allowed: 45 minutes　　Total marks: 400

1. 若干個正整數之和是 20，求它們乘積的最大可能值。 (15 分)
The sum of several positive numbers is 20, find their largest possible product. (15 marks)

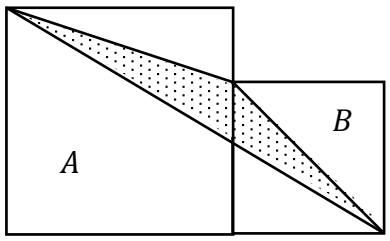
2. 在 2018 年，彼得 7 歲、他的弟弟 5 歲，彼得的媽媽和爸爸分別是 29 歲及 31 歲，他們四人的歲數都是質數。問最少多少年後，他們四人的歲數再次全都是質數？ (17 分)
In 2018, Peter was 7 years old, his brother was 5 years old. His mother and father were 29 years old and 31 years old respectively. The ages of all family members were prime numbers. At least how many years later will the ages of all these four persons be prime numbers again? (17 marks)

3. 計算 $99+98+98+97+97+97+96+96+96+96+\cdots+1+\cdots+1$。 (20 分)
Evaluate $99+98+98+97+97+97+96+96+96+96+\cdots+1+\cdots+1$. (20 marks)

4. 在數列 1、2、3、4、5、6、7、8、…中刪除 2、3 或 5 的倍數後，便會組成的新數列 1、7、11、13、17、19、…。求新數列中第 60 項的數值。 (20 分)
A new sequence 1, 7, 11, 13, 17, 19, … is formed by deleting the multiples of 2, 3 or 5 from the sequence 1, 2, 3, 4, 5, 6, 7, 8, … . Find the value of the 60^{th} term in the new sequence. (20 marks)

5. 米袋 X 重 10kg，其中 80%為白米、20%為紅米。米袋 Y 重 a kg，其中 60%為白米、40%為紅米。若米袋 X 和米袋 Y 的米混合，使混合後的米最少 36%為紅米，求 a 的最小值。 (23 分)
The weight of rice bag X is 10 kg. X contains 80% white rice and 20% red rice. The weight of rice bag Y is a kg. Y contains 60% white rice and 40% red rice. The rice in bag X and Y is mixed, find the minimum value of a if there is at least 36% of the mixed rice is red rice. (23 marks)

6. 在圖中，A 及 B 分別為兩個邊長 12 cm 及 8 cm 的正方形，求陰影部份的面積。 (24 分)
In the figure, A and B are two squares with sides 12 cm and 8 cm respectively. Find the area of the shaded region. (24 marks)



7. 一排長椅共有 90 個座位，其中有部份座位已經有人就座。這時，陳先生希望坐在這排長椅上，他無論坐在哪個座位上都與已就座的人相鄰。問原來至少有多少人已經就座？ (24 分)

There are 90 seats in a long bench. Some of the seats have already been occupied. Now Mr. Chan is going to sit on this bench. Interestingly, he will sit next to the person who has already been seated regardless of whichever seat he takes. At least how many persons have already been seated on the bench at first? (24 marks)

8. 若將一個兩位數的個位和十位換轉，數值將會相差 27。而個位和十位的和是 13，且十位大於個位，求該數。 (25 分)

If the units digit and the tens digit of a number are exchanged, the difference in value will be 27. It is given that the sum of the units digit and the tens digit of the number is 13 and the tens digit is greater than the units digit, find the number. (25 marks)

9. 計算 $(1-\frac{1}{2^2})(1-\frac{1}{3^2})(1-\frac{1}{4^2})\cdots(1-\frac{1}{20^2})$。 (25 分)

Evaluate $(1-\frac{1}{2^2})(1-\frac{1}{3^2})(1-\frac{1}{4^2})\cdots(1-\frac{1}{20^2})$. (25 marks)

10. 將 $\frac{14}{13}$ 化成小數，求它小數點後第 2019 位。 (26 分)

If $\frac{14}{13}$ is converted into decimal, find the 2019^{th} digit after the decimal point. (26 marks)

11. 在一個正方形的每邊加 1 個中間點(圖 1)，這些中間點可連成 6 條線。在一個正方形的每邊加 2 個中間點(圖 2)，這些中間點可連成 24 條線。若正方形的每邊加 3 個中間點(圖 3)，可連成多少條線？ (26 分)

If 1 middle point is added to each edge of the square (Figure 1), 6 lines can be formed by these middle points. If 2 middle points are added to each edge of the square (Figure 2), 24 lines can be formed by these middle points. If 3 middle points are added to each edge of the square (Figure 3), find the number of lines that can be formed. (26 marks)

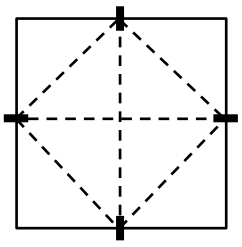


圖 1
Figure 1

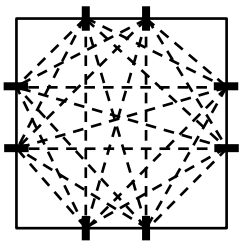


圖 2
Figure 2

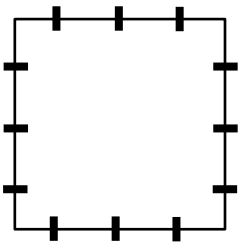
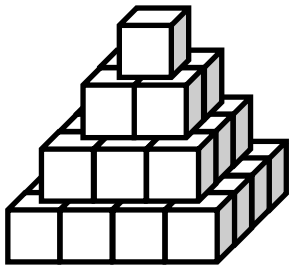
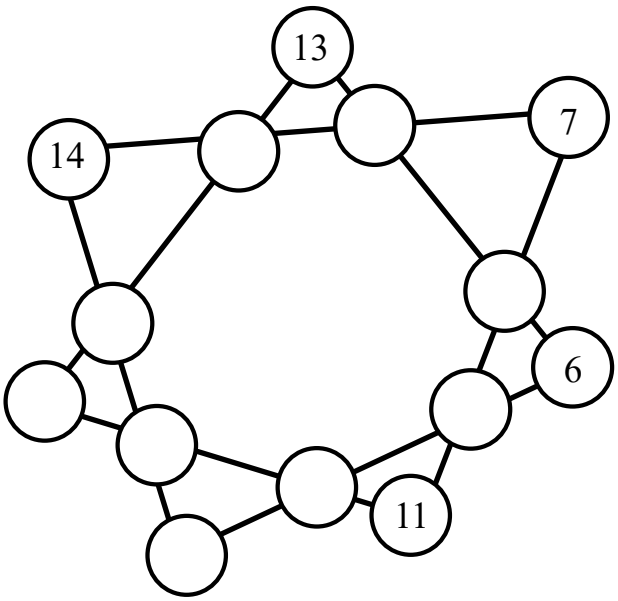


圖 3
Figure 3

12. 如圖，30 個邊長為 1 單位的正方體堆疊成一座四層塔，求這座塔的總表面面積。 (27 分)
As shown in the figure, 30 cubes of each side 1 unit are stacked together to form a four-level tower. Find the total surface area of the tower. (27 marks)

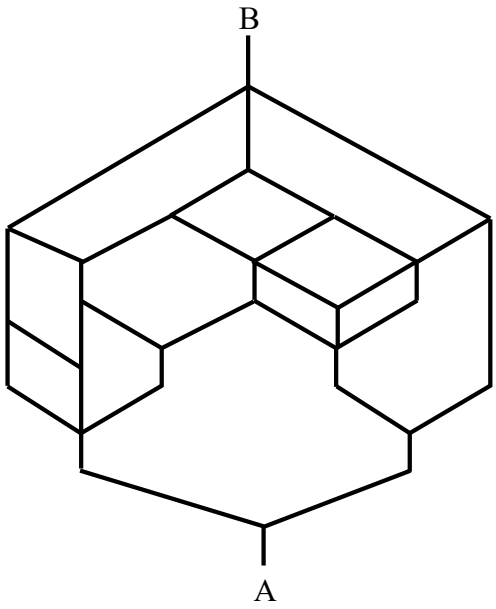


13. 將數字 1、2、3、4、5、8、9、10 和 12 填入空的圓形，使每條直線上圓形內數字之和是 30。(每個數字只能使用一次) (27 分)
Put numbers 1, 2, 3, 4, 5, 8, 9, 10 and 12 into the circles to make the sum of each straight line equals to 30. (Each number can only be used once) (27 marks)



14. 有 9 位美女，當中有說真話的天使，其餘都是說假話的魔鬼，
第一位美女說：「我們中間有 1 個魔鬼。」；第二位美女說：「我們中間有 2 個天使。」；
第三位美女說：「我們中間有 3 個魔鬼。」；第四位美女說：「我們中間有 4 個天使。」；
第五位美女說：「我們中間有 5 個魔鬼。」；第六位美女說：「我們中間有 6 個天使。」；
第七位美女說：「我們中間有 7 個魔鬼。」；第八位美女說：「我們中間有 8 個天使。」；
第九位美女說：「我們中間有 9 個魔鬼。」。
這些美女中有幾多個天使？ (32 分)
There are 9 beauties in total. Some is/are angel(s) who always tell(s) the truth, the rest is/are the devil(s) who always tell(s) lies.
The first beauty said, “There is one devil among us.”
The second beauty said, “There are two angels among us.”
The third beauty said, “There are three devils among us.”
The fourth beauty said, “There are four angels among us.”
The fifth beauty said, “There are five devils among us.”
The sixth beauty said, “There are six angels among us.”
The seventh beauty said, “There are seven devils among us.”
The eighth beauty said, “There are eight angels among us.”
The ninth beauty said, “There are nine devils among us.”
So how many angel(s) is/are there among these beauties? (32 marks)

15. 一隻螞蟻由 A 點住上走到 B 點，每到一個路口，牠都能選擇任何一個方向繼續向上走(牠只能向上走)，問由 A 點走到 B 點有多少條路徑？ (33 分)
An ant travels from point A to point B. Every time it reaches a junction, it can go upwards by any direction (it can only go upwards). Find the total number of different paths from *A* to *B*. (33 marks)



16. 一個填字遊戲需要依照下列規則：

規則一：將數字填入空格內，使數字能由 1 連續至 36。(圖一)

規則二：有交點「◆」的空格必須相連。(圖二)

規則三：最後填滿所有空格。(圖三)

A puzzle game follows the following rules:

Rule 1: Put the numbers from 1 to 36 to create a path of consecutive numbers. Numbers and links between cells are given to help finish the puzzle. Two consecutive numbers must be next to each other. (Figure 1)

Rule 2 : A link indicates a crossing point "◆" of the path. (Figure 2)

Rule 3 : At the end, the entire grid must be filled up. (Figure 3)

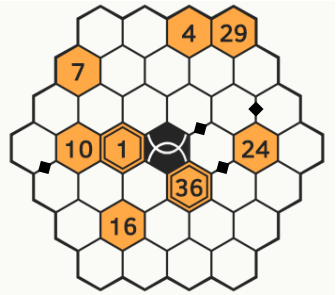


Figure 1
圖一

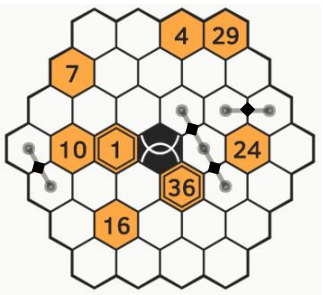
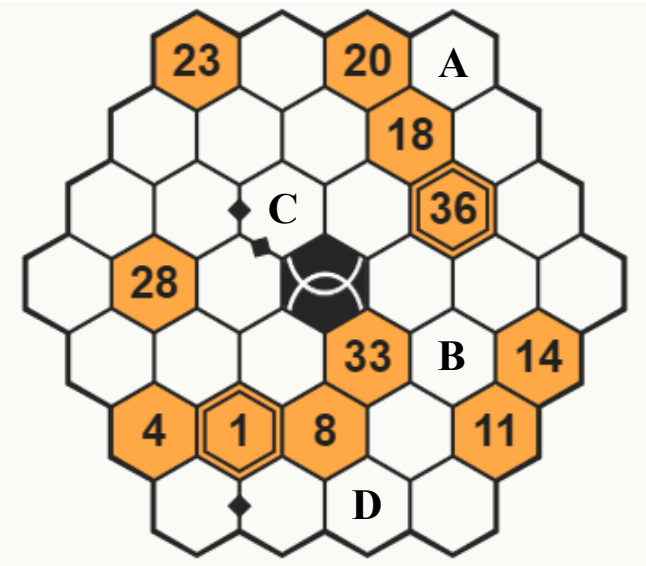


Figure 2
圖二

Figure 3
圖三

若要完成以下圖形，求 A、B、C 和 D 的值。 (36 分)

To complete the following puzzle, find the values of A, B, C and D. (36 marks)



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